Remark on "Conductance and Shot Noise for Particles with Exclusion Statistics" by Isakov, Martin, and Ouvry

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Isakov, Martin, and Ouvry [1] have recently proposed a fresh approach to the potential observation of fractional exclusion statistics. According to their argument, a clear signature of fractional statistics should exist in the shot noise of a Luttinger fluid, an ideal system postulated by some to underlie the well-founded Laughlin quasiparticle states of the two-dimensional electron gas. We elaborate on some delicate points made by Isakov *et al.* and reflect upon the relationship between this novel intuitive scheme and certain old issues of first principles.
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The interplay of high magnetic fields and many-body interactions in a two-dimensional electron gas produces the fractional quantum-Hall effect (FQHE) at low temperatures. At filling factor $\nu = 1/m$ (m is an odd integer) the Hall resistivity reaches a plateau, showing that the correlated quasiparticles carry fractional charge νe. The fractionally charged quasiparticles (FCQPs) *may* obey fractional exclusion statistics (FES) [2].

Recent shot-noise and Coulomb-blockade experiments confirm that one-dimensional FCQPs carry the edge-state currents in the FQHE, but cannot establish FES unambiguously. Isakov *et al.* [1] consider a novel theoretical possibility. They generalize the Landauer-Büttiker first-quantized method for one-dimensional conductance and shot noise to noninteracting particles with FES, thought to be relevant to the edge-state FQHE. The explicit formula obtained for the crossover of shot noise to thermal noise stretches its normal interpretation in terms of the noninteracting Schottky and Johnson-Nyquist formulas.

The results of Ref. [1] may be of experimental interest. Despite this, we note some potentially troublesome issues of principle. These warrant closer examination.

(1) To obtain a FES noise spectral density strictly by first quantization, one must first have a counting method going beyond the normal occupancies for fermions and bosons. The proposal of Isakov *et al.* fails to recover standard results for normal fluctuations, as required by second quantization [3]. To redeem their argument they must add an *ad hoc* term to their Eq. (3), maintaining presumed conformity with the fluctuation-dissipation theorem (FDT).

We draw attention to Isakov *et al.*'s stated need to call on the FDT with no idea whatever of how its microscopic proof plays out in this new physical context: a knowledge gap with serious implications. In every many-body system, the FDT is an absolutely essential structural link between equilibrium fluctuations and dissipative response. Its validity must be deduced from the model's axioms; it can never be established inductively. This means that while the microscopic fluctuations prescribe the observable conductance, the converse does not hold. The present approach is unable to derive the appropriate FDT. It allows only speculation on some FDT-like ansatz that is cut and fit, inductively, to one's notions of what the fluctuations might be (given the conductance). Such *ad hoc* inversion of the theorem's prescriptive meaning severely undermines the microscopic credibility of the state-counting argument.

(2) FCQPs carry the FQHE edge-state current in *one dimension*, and thus represent the excitations of a correlated Luttinger fluid. This is totally unlike a fluid of "independent" quasiparticles. There are no grounds, in second quantization, to describe exclusion statistics by an ansatz that interpolates intuitively between the limiting occupancies for *free* bosons and fermions. Not surprisingly, the oversimplified accounting leads again to a familiar difficulty: a term must be added *ad hoc* to keep faith with the FDT. For bosons a further unphysical feature appears, in that the zero-frequency spectral density for shot noise diverges. This dilemma is avoidable – in second quantization – via the generalized commutation relations introduced long ago by H. S. Green [4].

(3) The crossover of the spectral density from thermal to shot noise is derived by assuming $\exp(\beta\mu) \gg 1$. This occurs when either the thermal energy (temperature) β^{-1} is very small, or the chemical potential μ is very large. The latter corresponds to a high-density system. If the FCQPs are truly independent, the system will be ballistic. In the quantum ballistic limit there is no shot noise.

Shot noise is inherently a nonequilibrium phenomenon; any *correlations* in it must involve nonlinearity in the applied field. For normal fermions, linear theories based on quantum-coherent (or on semiclassical) diffusion imply a smooth crossover from low temperatures and high fields, to high temperatures and low fields. While this has strong appeal in making sense of experiments, a first-principles nonequilibrium theory is not in sight.

The Landauer-Büttiker approach is akin to Kubo linear response. For shot noise in correlated systems, a major question should be how its suppression is modified by many-body interactions. In a strictly single-particle, linear-transport picture, suppression enters via the factor $T(1-T)$ where T is the one-particle transmission probability through the system. In both limits $T \to 0$ and $T \to 1$, shot noise tends to zero. For $T \to 0$ the system is nonconducting; trivially, there is no shot noise. When $T \to 1$, the system is ballistic; again, within strictly first-quantized treatments of fluctuations, there is no shot noise. However, as the quasiparticles of a conventional Fermi system *do* interact, it is fair to wonder whether their shot noise is indeed so comprehensively suppressed.

One is now asked to go beyond the normal, and to postulate fractional statistics for (nearly free) quasiparticles in the Luttinger liquid, an utterly correlation-dominated and certainly unconventional many-body system. Isakov, Martin, and Ouvry [1] raise many more open problems than have been answered satisfactorily for noise in the Landau model of normal quasiparticles. Furthermore, their concept of correlated quasiparticle systems amounts to the following.

- Strongly correlated systems can always be renormalized into an ensemble of quasiparticles. These may possess fractional statistics (which can be intuited).
- Since the low-lying excitation spectrum of the system will resemble one-body behavior, one can suppose that the residual quasiparticle interactions are literally weak. The system is effectively free.
- One can then safely analyze the two-body fluctuations (noise) of such systems as if the quasiparticles did not actually interact. Said differently: the mean-square fluctuations of a correlated system are fully and uniquely given by the effective mean single-particle properties.

In our view, such a proposal hardly fits what is known even for normal Fermi liquids. One need only recall that the low-order Landau parameters (essentially associated with quasiparticle renormalization) are quantitatively locked into all the higher-order ones (associated with the interactions *among* the quasiparticles) by a standard sum rule [5]. This basic constraint will draw in higher and higher angular moments of the "residual" quasiparticle interaction as the coupling strength in the underlying Hamiltonian is increased. No-one is yet sure, of course, what holds for the Luttinger case.

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